Analysis and Synthesis Sparse Representation Models for Image Modeling

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Outline

- Introduction
  - Image Restoration and Enhancement
  - Synthesis & Analysis Sparsity Models

- Convolutional sparse coding for image super-resolution
  - Convolutional Sparse Coding v.s. Sparse Coding
  - The Proposed Method

- Guided Image Enhancement via Weighted Analysis Sparsity Model
  - Dependency Modeling for Guided Enhancement
  - Learning dynamic guidance for guided depth enhancement

- Ongoing and Future Works
  - Image Separation without Training Data
  - Image Restoration with Deep Denoisers
  - Optimization Inspired Network Structure Design
Introduction
Image restoration and enhancement problems

Image Denoising
\[ y = x + n \]

Image Deconvolution
\[ y = k \otimes x + n \]

Contrast Enhancement
\[ y = M \circledast x + n \]

Image Super-resolution
\[ y = D (k \otimes x) + n \]

Image Inpainting
\[ y = M \odot x + n \]

...
Synthesis & Analysis sparsity models

● Synthesis representation models

Synthesis based sparse representation model assumes that a signal $x$ can be represented as a linear combination of a small number of atoms chosen out of a dictionary $D$:

$$x = D\alpha, \text{ s.t. } \|\alpha\|_0 < \varepsilon$$

A dense solution

A sparse solution
Synthesis & Analysis sparsity models

**Synthesis representation models**

Synthesis based sparse representation model assumes that a signal $x$ can be represented as a linear combination of a small number of atoms chosen out of a dictionary $D$:
Synthesis & Analysis sparsity models

- **Analysis representation models**

Analysis model generate representation coefficients by a simple multiplication operation, and assumes the coefficients are sparse:

\[ \|Px\|_0 < \varepsilon \]
Synthesis & Analysis sparsity models

● Analysis representation models

Analysis model generate representation coefficients by a simple multiplication operation, and assumes the coefficients are sparse:
Synthesis & Analysis sparsity models

**Synthesis model**

\[ \min_{\alpha} \frac{1}{2} \| y - D\alpha \|_F^2 + \psi(\alpha) \]
\[ x = D\alpha \]

- Representative methods
  - KSVD, BM3D, LSSC, NCSR, et. al.

- **Pros**
  - Synthesis model can be more sparse

- **Cons**
  - Patch prior modeling needs aggregation
  - Time consuming

**Analysis model**

\[ \min_{x} \frac{1}{2} \| y - x \|_F^2 + \phi(Px) \]

- Representative methods
  - TV, wavelet methods, FRAME, FOE, CSF, TRD et. al.

- **Pros**
  - Patch divide free
  - Efficient in the inference phase

- **Cons**
  - Not as sparse as synthesis model, limited capacity in modeling image prior.
Convolutional sparse coding for image super-resolution
Convolutional Sparse Coding v.s. Sparse Coding Coding

Consistency constraint
Convolutinal Sparse Coding v.s. Sparse Coding

Sparse coding

$$\min_{\alpha} \| y - D\alpha \|^2_F + \phi(\alpha)$$

Convolutional sparse coding

$$\min_{\mathbf{z}} \| \mathbf{Y} - \sum f_i \otimes z_i \|^2_F + \sum \phi(z_i)$$

Matrix Form
Convolutional sparse coding for image SR

The Training Phase
- LR Filter Learning
- Joint HR Filter and Mapping Function Learning
- \( N \) LR feature maps
- \( M \) HR feature maps
- CSC LR Filter Learning
- Mapp. Func. Learning
- HR Filter Learning

The Testing Phase
- \( N \) LR filters
- Mapp. Func.
- \( M \) HR filters
- HR Feature Map Estimation
- Convolution
Convolutional sparse coding for image SR

The Training Phase

- LR Filter Learning
- Joint HR Filter and Mapping Function Learning

\[ \min_{Z, f} \| Y - \sum_{i=1}^{N} f^l_i \otimes Z^l_i \|_F^2 + \lambda \sum_{i=1}^{N} \| Z^l_i \|_1 \]

\[ s.t. \| f^l_i \|_F^2 \leq 1 \]
Convolutional sparse coding for image SR

\[
\{ f^h, W \} = \min_{f, W} \| X - \sum_{j=1}^{M} f^h_j \otimes g(Z^l_j; w_j) \|_F^2,
\]

\[s.t. \quad \| f^h_j \|_F^2 \leq \epsilon; \quad w_j \geq 0, \quad |w_j|_1 = 1\]
Convolutional sparse coding for image SR

The Training Phase

N LR filters

Mapp. Func.

HR Feature Map Estimation

The Testing Phase

M HR filters

Convolution
Convolutional sparse coding for image SR

Optimization: SA-ADMM

\[
\{W\} = \arg \min_W \sum_{k=1}^K \|X_k - \sum_{j=1}^M f^h_j \otimes g(Z_{k,j}^l; w_j)\|_F^2, \quad \text{s.t. } w_j \geq 0, |w_j|_1 = 1.
\]

Denote by \(\tilde{Z}_i^l\) the upsampling of LR feature map

\[
\tilde{Z}_{k,i}^l(x', y') = \begin{cases} 
Z_{k,i}^l(x, y) & \text{if } \text{mod}(x', \text{factor}) = 0 \text{ and } \text{mod}(y', \text{factor}) = 0 \\
0 & \text{otherwise}
\end{cases},
\]

then we have

\[
[\text{vec}(Z_{k,1}^h), \text{vec}(Z_{k,2}^h), \ldots, \text{vec}(Z_{k,M}^h)] = [\text{vec}(\tilde{Z}_{k,1}^l), \text{vec}(\tilde{Z}_{k,2}^l), \ldots, \text{vec}(\tilde{Z}_{k,N}^l)] \ast W,
\]

The original problem can be write as:

\[
\{W\} = \sum_{k=1}^K \arg \min_W \|\text{vec}(X) - [F_1^h, \ldots, F_M^h] \ast \begin{bmatrix}
[\text{vec}(\tilde{Z}_{k,1}^l), \ldots, \text{vec}(\tilde{Z}_{k,N}^l)] \\
\vdots \\
[\text{vec}(\tilde{Z}_{k,1}^l), \ldots, \text{vec}(\tilde{Z}_{k,N}^l)]
\end{bmatrix}\|_F^2, \\
\text{s.t. } w_j \geq 0, |w_j|_1 = 1.
\]
Convolutional sparse coding for image SR

Optimization: SA-ADMM

\[
\{\mathbf{W}\} = \sum_{k=1}^{K} \arg \min_{\mathbf{W}} \| \text{vec}(\mathbf{X}) - [\mathbf{F}_1^h, \ldots, \mathbf{F}_M^h] ^* \begin{bmatrix} \text{vec}(\mathbf{\tilde{Z}}_{k,1}^l), \ldots, \text{vec}(\mathbf{\tilde{Z}}_{k,N}^l) \\ \vdots \\ \text{vec}(\mathbf{\tilde{Z}}_{k,1}^l), \ldots, \text{vec}(\mathbf{\tilde{Z}}_{k,N}^l) \end{bmatrix} \|_F^2 \\
\text{s.t. } w_j \geq 0, |w_j|_1 = 1.
\]

\[
\{\mathbf{W}\} = \sum_{k=1}^{K} \arg \min_{\mathbf{W}} \| \text{vec}(\mathbf{X}) - \mathbf{A} \ast \text{vec}(\mathbf{W}) \|_F^2 \\
\text{s.t. } w_j \geq 0, |w_j|_1 = 1.
\]

SA-ADMM

\[
\text{vec}(\mathbf{W})_{t+1} = [L \text{vec}(\mathbf{W})_t - \rho(T_t - S_t) - \frac{1}{K} \sum_{k=1}^{K} A_k^T(A_k \text{vec}(\mathbf{W}_{\tau_j(t)}) - X_k)]/(\rho + L)
\]

\[
S_{t+1} = \arg \min_{S} \frac{\rho}{2} \| \mathbf{W}_{t+1} + T_t - S \|_2^2, \quad \text{s.t. } s_j \geq 0, \sum s_j = 1
\]

\[
T_{t+1} = T_t + \mathbf{W}_{t+1} - S_{t+1}
\]
Convolutional sparse coding for image SR
Convolutional sparse coding for image SR
Guided Image Enhancement via Weighted Analysis Sparsity
Dependency Modeling

- Dependent image data
  - MRI
  - PET
  - Depth
  - RGB

- Guided enhancement

Single Image Enhancement

Guided Image Enhancement
Dependency Modeling

• Previous Arts
  – 1\textsuperscript{st} order method: co-different
    \[
    \sum_i \sum_{j \in S(i)} (x_i - x_j)^2 \varphi(g_i - g_j)
    \]
  – 2\textsuperscript{nd} order method: TGV
  – Other priors: Non-local mean
  – Data-driven method: joint dictionary learning
Dependency Modeling

• Weighted Analysis Sparse Representation Model

\[
\sum_i \sum_{j \in S(i)} (x_i - x_j)^2 \varphi(g_i - g_j)
\]

\[
\sum_i \sum_{j \in S(i)} (1 - \rho(x_i - x_j)) \varphi(g_i - g_j)
\]

Generalize the model: from one-order point-wise relationship to high-order local prior.

\[
\hat{x} = \arg\min_x f(x, y) + \sum \rho(k_x^i \ast x) \otimes \varphi(k_g^i \ast g)
\]

• How to choice \(\rho, \varphi, k\)?
• Highly non-convex, hard to optimize!
Guided image enhancement via weighted analysis

sparsity

- Task-driven training of stage-wise parameters
  - Solving weighted analysis sparse representation model with gradient descent, we have:

  \[
  \hat{x} = \arg\min_x f(x, y) + \sum \rho(k^i_x \ast x) \odot \varphi(k^i_g \ast g)
  \]

  \[
  x^{t+1} = x^t - \tau \left( \Delta f(x^t, y) + \sum k^i_x \rho T \rho' (k^i_x \ast x) \odot \varphi(k^i_g \ast g) \right)
  \]

- Stage-wise parameter training

  \[
  \min_{k_x, k_g, \rho, \varphi} \text{loss}(x^{t+1}(x^t; k_x, k_g, \rho, \varphi) - x^{gt})
  \]

  \[
  \text{s.t. } x^{t+1}(x^t; k_x, k_g, \rho, \varphi) = x^t - \tau \left( \Delta f(x^t, y) + \sum k^i_x \rho T \rho' (k^i_x \ast x) \odot \varphi(k^i_g \ast g) \right)
  \]
Guided image enhancement via weighted analysis via sparsity

Initialization $x^0$

Convolution $g$

Nonlinearity

Pointwise Multiplication

Fidelity term

Regularization term

$\sum$

Enhanced depth map

Low quality depth map

High quality color image

$\sigma$

$x^1$
Guided image enhancement via weighted analysis

Sparsity
Guided Image Enhancement via Weighted Analysis Sparsity

- Experimental results
Guided Image Enhancement via Weighted Analysis Sparsity

- Experimental results
Ongoing and Future Works
Image Separation without Training Data

• Complementary Property of ASR and SSR

• Layer Separation

\[ \min_{u,v} f(y - u - v) + \rho_s(u) + \rho_A(v) \]
Image Restoration with Deep Denoisers

• Half Quadratic Splitting

\[ \min_x f(y, x) + \rho(x) \]

Half Quadratic Splitting

\[ \min_{x,s} f(y, x) + \lambda \|x - s\|_F^2 + \rho(s) \]

\[ \min_x f(y, x) + \lambda \|x - s\|_F^2 \]

\[ \min_s \lambda \|x - s\|_F^2 + \rho(s) \]

Deep Denoiser
Optimization Inspired Network Structure Design

• State-of-the-art Performance Has been Achieved by Deep Models
  – Non-blind Super-Resolution
  – Gaussian Denoising
  – Non-blind Deblur

• More Complex Restoration problems
  – Blind Deblur, SR, Denoising?
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THANKS!